Holyport CE Primary School & Foundation Unit

Mathematics Calculation Policy

Introduction

This calculation policy has been devised to support staff in understanding both the expectations for fluency of the 2014 curriculum and the progression of calculation concepts through a child's mathematical development. This policy has been written using guidance from the White Rose Maths Hub whose materials the school uses in their planning and delivery of mathematics lessons.

Principles

- This calculation policy is focused on developing proficiency with the expected formal methods by the end of Year 6 and hence the progression guidance provided for each operation is designed to flow into the expected method as exemplified on the National Curriculum Appendix document.

- This policy reflects the school's approach to a mastery curriculum through a Concrete, Pictorial and Abstract approach (CPA) which all staff are expected to follow.

- Specific practical equipment and approaches have been suggested for each objective and strategy to support children in developing the conceptual understanding that will enable them to move more rapidly and efficiently towards the formal written methods expected.

- It is expected that pupils will have access to practical resources, to support them in developing conceptual understanding of an operation, regardless of the year group they are in.

- It is expected that all pupils will be encouraged to use pictorial strategies to support their calculations, regardless of the year group they are in. This includes bar modelling and other visual representations.

- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach.

- Teachers should ensure that they use the correct mathematical vocabulary particularly when modelling formal written methods. A glossary of terms is included at the end of this policy.

- Teachers should help pupils develop efficient mental calculation strategies e.g. through starters/quick fire quizzes and throughout the lesson.

- The school uses approaches such as odd one out, true or false, always, sometimes never – so that pupils develop their reasoning and can identify misconceptions in their own understanding.

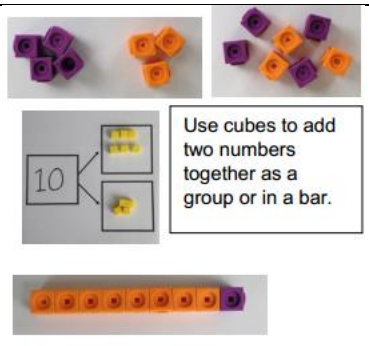
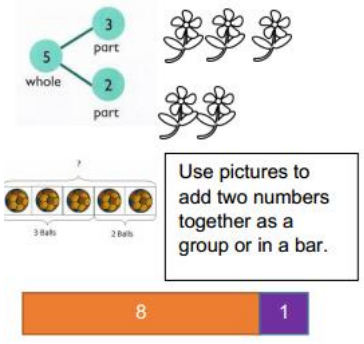
Expectations and Progression in Calculations per year group

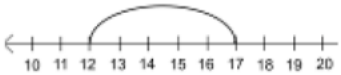
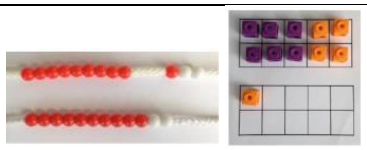
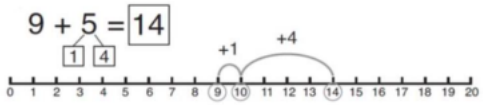
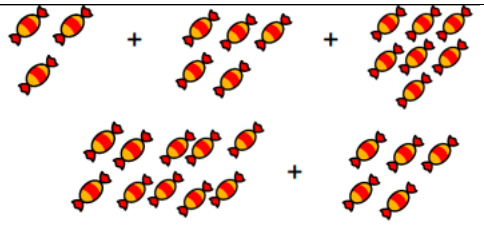
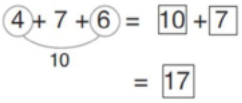
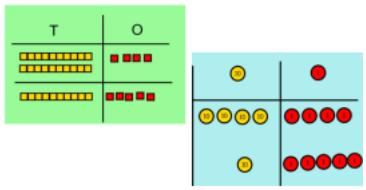
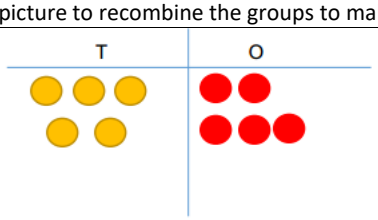
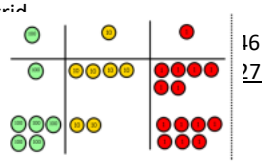
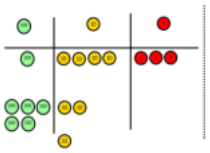
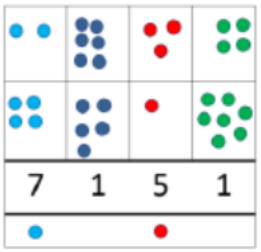
Please refer to this, when looking at the CPA strategies that follow this table:

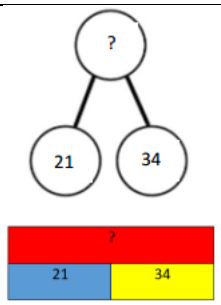
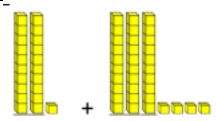
	Foundation Stage	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Add single digit numbers using practical activities	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping	Column method – regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals – with the same amount of decimal places)	Column method with regrouping. (Decimals – with different amounts of decimal places)
Subtraction	Subtract single digit numbers using practical activities	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back. Find the difference. Part whole model. Make 10. Column method – no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals – with the same amount of decimal places)	Column method with regrouping. (Decimals – with different amounts of decimal places)
Multiplication	Problem solve using doubling	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays – showing commutative multiplication	Counting in multiples Repeated addition Arrays – showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Problem solve using halving and sharing	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit – concrete and pictorial)	Division within arrays Division with a remainder Short division (2 digits by 1 digit – concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number – interpret remainders as whole numbers, fractions or round)

Progression in calculations

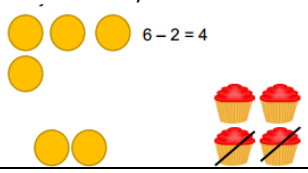
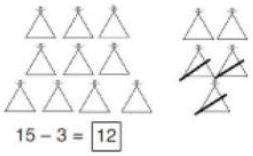
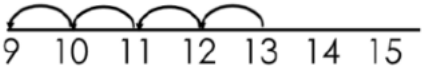
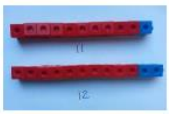
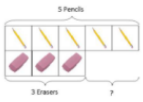
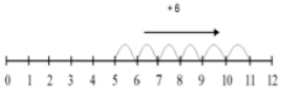
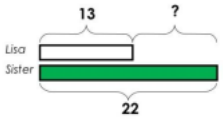
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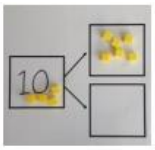
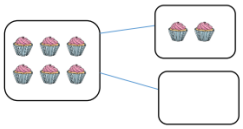
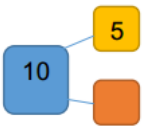
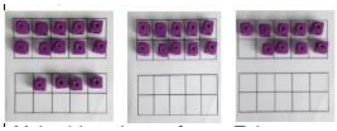
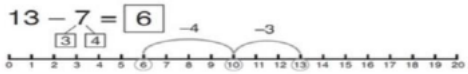
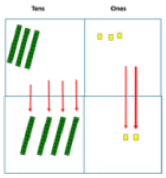
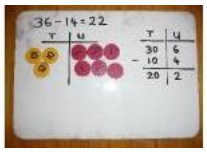
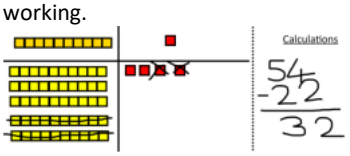
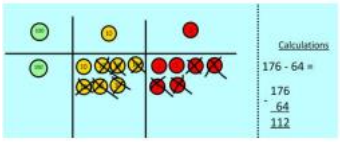
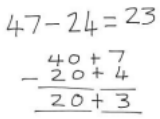
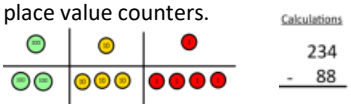
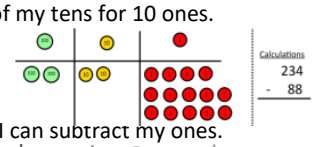
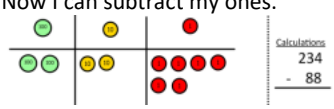
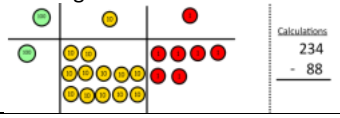
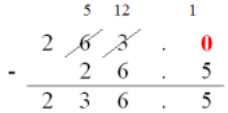
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part whole model.			4 + 3 = 7 10 = 6 + 4 Use the part-part whole diagram to move into the abstract.

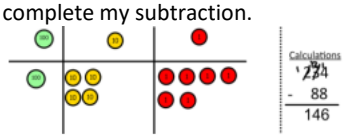
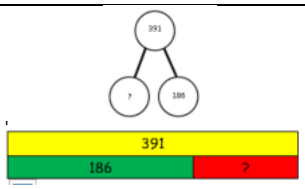
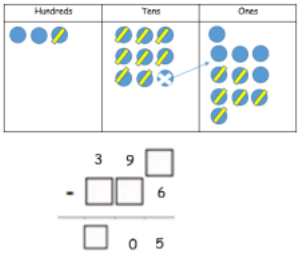
Objective and Strategies	Concrete	Pictorial	Abstract
Starting at the bigger number and counting on.	 Start with the larger number on the bead string and then count on the smaller number 1 by 1 to find the answer	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	Use pictures or a number line. Regroup or partition the smaller number to make 10.  3 + 9 = 9 + 5 = 14 1 4 +1 +4 	7 + 4 = 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?
Adding three single digits.	4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7  Following on from making 10, make 10 with 2 of the digits (if possible) and then add on the third digit.	 3 + 5 + 7 = 15 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers to make 10 and then add on the remainder.
Column method – no regrouping.	24 + 15 =  Add together the ones first then the tens. Use the Base 10 blocks first before moving onto place value counters.	 After practically using the Base 10 blocks and place value counters, children can draw the counters to help them to solve additions.	21 + 42 = 21 + 42
Column method – regrouping.	Make both numbers on a place value grid  Add up the units and exchange 10 ones for one ten.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

	This can also be done with Base 10 to help the children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move onto decimals, money and decimal place value counters can be used to support learning.		72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2
Fluency variation, different ways to ask children to solve 21 + 34:			
	Sam saved £21 one week and £34 another. How much did he save in total?	21 + 34 21 + 34 = ? = 21 + 34 What's the sum of twenty one and thirty four?	 Use missing digit problems too: 

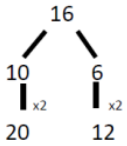
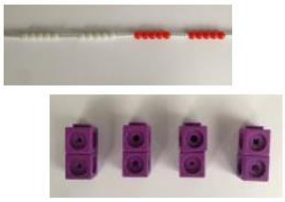
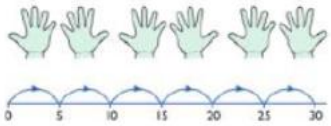
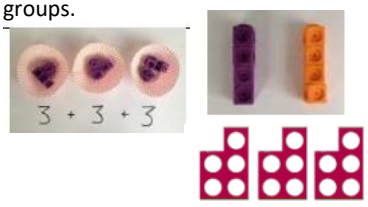
Subtraction

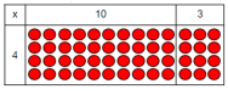
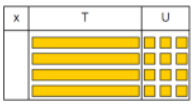
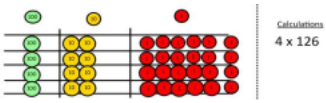
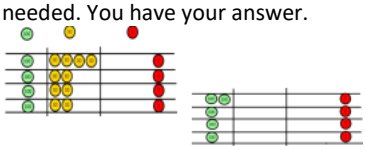
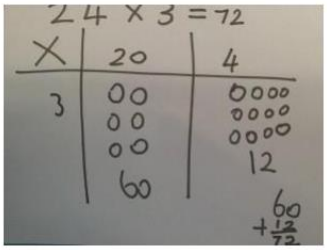
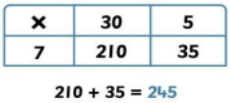
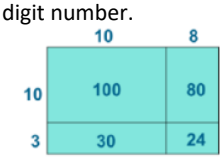
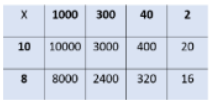
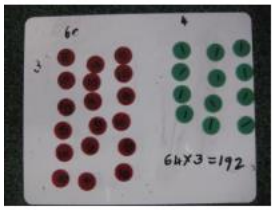
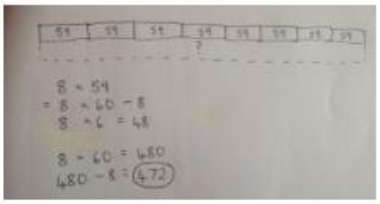
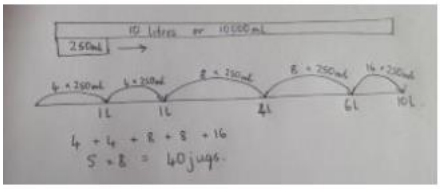
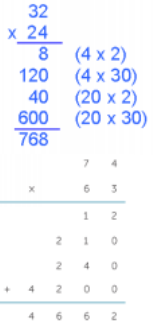
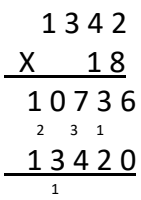
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc. to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away. 	18 – 3 = 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line  This can progress all the way to counting back using 2 digit numbers.	Put 13 in your head and count back 4. What number are you at? Use your fingers to help.
Find the difference	Compare amounts and objects to find the difference.  e cubes to ild towers to d the ference. Use basic bar models with items to find the difference. 	Count on to find the difference.  Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches and Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Objective and Strategies	Concrete	Pictorial	Abstract
Part whole model	Link to addition – use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts what is the other part? 	Use a pictorial representation of objects to show the part whole model. 	Move to using numbers within the part whole model. 
Make 10	14 – 9 =  Make 14 on the tens frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off? What number do we end up with?
Column method without regrouping	Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first. 	Draw the Base 10 or place value counters alongside the written calculation to help show working.  	 This will lead to a clear written column subtraction: 32 – 12 20
Column method with regrouping	Use Base 10 to start with before moving onto place value counters. Start with one exchange before moving onto 2 exchanges. Make the larger number with the place value counters.  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for 10 ones.  Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange on hundred for 10 tens. 	Draw the ocunters onto a place value grid nad show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchanging/regrouping.  Just writing the numbers as shown above shows that the child understands the method and knows when to exchange/regroup.	Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method  This will lead to an understanding of subtracting any number including decimals 

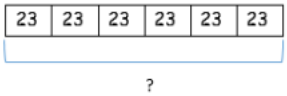
	Now I can take away 8 tens and complete my subtraction.  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		
Fluency variation, different ways to ask children to solve 391 - 186:			
	Tim spent £391. Timmy spent £186. How much more did Tim spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	391 – 186 ? = 391 – 186 391 – 186 Find the difference between 391 and 186. Subtract 186 from 391 What is 186 less than 391?	What's the calculation? What's the answer? 

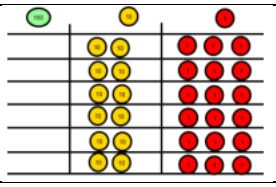
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number  Double 4 is 8 4 x 2 = 8	Use pictures to show ho to double a number. Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count in multiples supported by concrete objects in equal groups. 	Use a number line of pictures to continue support in counting in mulitples. 	Count in multiples of a number out loud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Repeated addition	Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star bscuits on. How many biscuits are there? 	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays – showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. 	Draw arrays in different rotations to find **commutative** multiplication sentences. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 x 3 = 15 3 x 5 = 15 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15

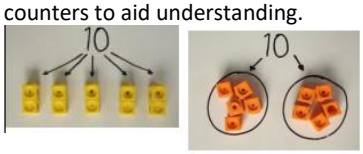
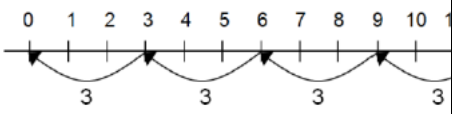
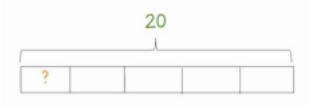
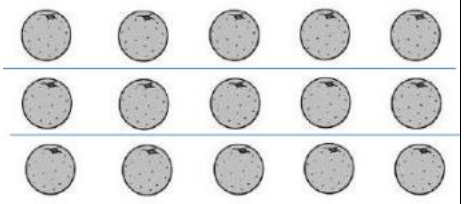
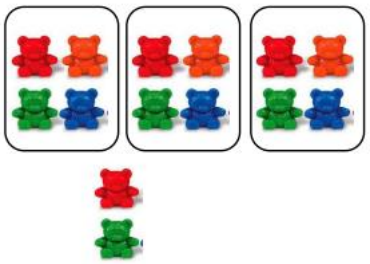
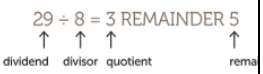
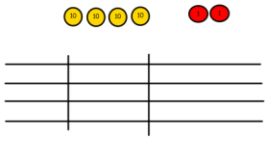
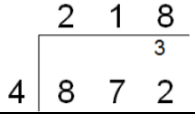
Grid method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method. Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126  Add up each column starting with the ones making any exchanges needed. You have your answer. 	Children can represent the work they have done with place value counters n a way that they understand. They can draw the counters, using colours to show different amounts or just use cirlces in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  You can then use the grid method to multiply by a 2 digit number.  
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer.  This moves to the more compact method. 

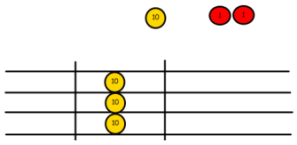
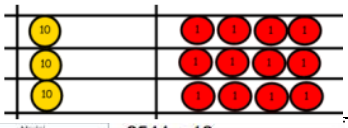
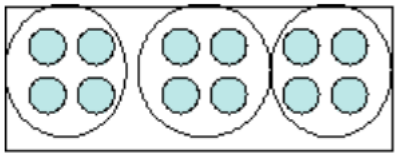
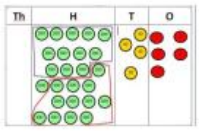
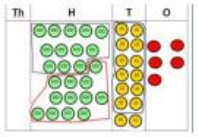
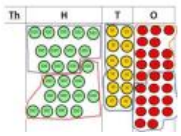
Fluency variation, different ways to ask children to solve 6 x 23:

 With the counters, prove that 6 x 23 = 138	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?	Find the product of 6 and 23. 6 x 23 = ? = 6 x 23	What's the calculation? What's the answer?

Why is 6 x 23 = 23 x 6	Tom saved 23p three days a week. How much did he save in 2 weeks?	6 23 x 23 x 6	

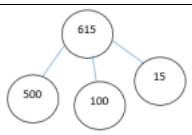
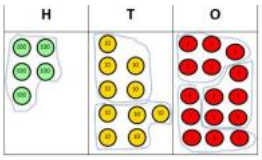
Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created  e.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 7 = 4
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line see how many more you need to jump to find a remainder  Draw dots and group them to divide an amount and clearly show a remainder 	Complete written divisions and show the remainder using r 
Short division	Use place value counters to divide using the bus stop method alongside. 42 ÷ 3 = 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.	Begin with divisions that divide equally with no remainder 

	Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for 10 ones and then share the ones equally among the groups. We look at how much is in 1 group so the answer is 14. 	 Encourage them to move towards counting in multiples to divide more efficiently.	Move onto divisions with a remainder $\begin{array}{r} 8\ 6\ r\ 2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad\ 3 \end{array}$ Finally move into decimal places to divide the total accurately $\begin{array}{r} 1\ 4\ .\ 6 \\ 3\ 5\overline{)\,5\ 1\ 1\ .\ 0} \\ \quad\ 16\ \ 21 \end{array}$
Long division	 2544 ÷ 12 How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds.  $12\overline{)2544}$ 0 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  $12\overline{)2544}$ 02, 24, 1 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2  $12\overline{)2544}$ 021, 24, 14, 12, 2 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2  $12\overline{)2544}$ 0212, 24, 14, 12, 24, 24, 0	Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books. Use this method to explain what is happening and as soon as they have understood that move on to the abstract method as this can be a time consuming process.	$12\overline{)2544}$ 0 Step one – exchange 2 thousand for 20 hundreds so we now have 25 hundreds. $12\overline{)2544}$ 02, 24, 1 Step two – How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. $12\overline{)2544}$ 021, 24, 14, 12, 2 Step three – exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I have grouped and the 2 is how many tens I have left. $12\overline{)2544}$ 0212, 24, 14, 12, 24, 24, 0 Step four – exchange the 2 tens for 20 ones. The top 24 shows how many ones I have, the bottom 24 is how many ones I have grouped and the 0 is what I have left. So 2544 ÷ 12 = 212

Fluency variation, different ways to ask children to solve 615 ÷ 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?	I have £615 and share it equally	$5\overline{)615}$	What's the calculation? What's the answer?

	between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	615 ÷ 5 = ? = 615 ÷ 5 How many 5s go into 615?	

Glossary of Terms

2-digit number– a number with 2 digits like 23, 45, 12 or 60

3-digit number – a number with 3 digits like 123, 542, 903 or 561

Addition facts – knowing all number facts not just those that total 10 or 20. For example knowing that 1 + 1 = 2 and 1 + 3 = 4 and 2 + 5 = 7 etc.

Array - An array is an arrangement of a set of numbers or objects in rows and columns – it is mostly used to show how you can group objects for repeated addition or subtraction.

Bead String/Bar – a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting 'spaces' rather than objects.

Bridging – when a calculation causes you to cross a 'ten boundary' or a 'hundred boundary' e.g. 85 + 18 will bridge 100.

Compact vertical – the name of the recommended written method for addition whereby the numbers are added in columns, 1s first then 10s and so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line which is sometimes referred to as 'carrying'.

Concrete apparatus – objects to help children count and calculate– these are most often cubes (multilink) but can be anything they can hold and move including Cuisenaire rods, Dienes rods (hundreds, tens and units blocks), straws, Numicon, Place Value counters etc.

Count all – when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get **16**.

Count on/back – when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12, 13, 14, 15, **16**

Decimal number – a number with a decimal point e.g. 2.34 (said as two point three four)

Decomposition – the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1s to facilitate this. This is the traditional 'borrowing' form of column method, which is different to the 'payback' method.

Dienes Rods (or Base 10) – this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1s, 10s, 100s and 1000s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to 'exchange' them for smaller or larger blocks where necessary.

Difference – the gap between numbers that is found by subtraction e.g. 7-5 can be read as '7 take away 5' or as the 'difference between 7 and 5'

Dividend – the number being divided in a calculation

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2

Efficient Methods – the method(s) that will solve the calculation most rapidly and easily

Equals - is worth the same as (be careful not to emphasise the use of = to show the answer)

Exchanging – Swapping a '10' for ten '1s' or a '100' for ten '10s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column

Factor – a number that divides exactly into another number, without remainder

Grid method – a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping – an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts

Halve – divide a number by 2

Integer - a whole number (i.e. one with no decimal point)

Inverse – the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts – times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know 4x7 then you also know 40 x 70, 4 x 0.7 etc.

Long Division – formal written of division where the remainders are calculated in writing each time (extended version of short division)

Long Multiplication – formal written method of column multiplication

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number

Number bonds – 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100

Number line – a line either with numbers or without (a blank numberline).
The number line emphasises the continuous nature of numbers and the existence of 'in-between' numbers that are not whole. It is based around the gaps between numbers.
Children use this tool to help them count on or count back for addition of subtraction. As they get older, children will count in 'jumps' on a number line e.g. to add 142 to a number they may 'jump' 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.

Number track – a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon – practical maths equipment that teaches children the names and values of numbers 1-10 initially but them helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.

One-Step Calculation – a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.

Partition – split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2

Place Value – the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567

Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, then add the units to that total

Remainder – a whole number left over after a division calculation

Repeated addition – repeatedly adding groups of the same size for multiplication

Sharing – an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)

Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called 'bus stop')

Significant digit – the digit in a number with the largest value e.g. in 34 the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'

Sum – the total of two or more numbers (it implies addition). Sum should not be used as a synonym for calculation.

Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.

HOLYPORT CE PRIMARY SCHOOL & FOUNDATION UNIT	
Date policy adopted and signed by LGB: July 2022	Name and signature of Governor/ Chair of Governors:
Name of Head teacher: Amanda Featherstone	Signature of Head teacher:
Date to be reviewed:	This policy will be subject to review every three years by the Governing Body or more frequently in the light of changes in legislation or advice from the ODST/LA.